HYDRAULICS

LECTURE 3: OPEN CHANNEL FLOW

PE REVIEW COURSE
PENN STATE BEAVER CONTINUING EDUCATION

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OUTLINE

- Chapter 19: Open Channel Flow
  - Basics
  - Flow types
  - Uniform flow (Manning's Equation)
  - Hydraulic jump
  - Example problems

OPEN CHANNEL

- A fluid passageway that allows part of the fluid to be exposed to the atmosphere.
- Examples:
  - Natural waterways (rivers, streams, etc.)
  - Canals (man-made channels)
  - Culverts
  - Pipes with gravity flow (e.g., sewers)

CHANNELIZED PORTION OF THE TURTLE CREEK

TYPES OF FLOW

- Flow is a function of location and time
- Two types of flows

1. Uniform Flow: Flow depth does not change along the length of the channel
2. Steady Flow: Flow rate (volume per unit time) at a given location doe not change with time.

OPEN CHANNEL FLOW PARAMETERS

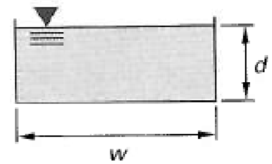
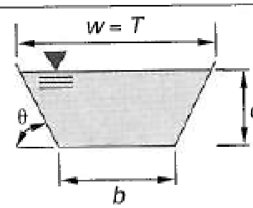
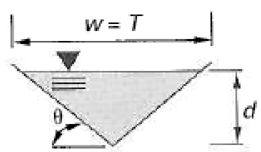
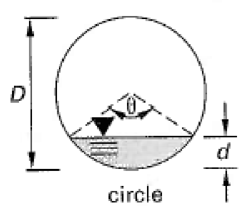
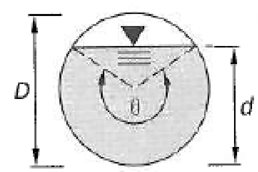
1. Flow rate $Q = A \times v$ … 19.1
   A = flow area
   v = mean velocity
2. Hydraulic radius $R = A / P$ … 19.2
   P = wetted perimeter = D/4 for circular pipes
3. Hydraulic depth $D_h = A / w$ … 19.3
   w = channel width at fluid surface (top width)

- Table 19.2: A, P, and R formulae for various channel shapes

OPEN CHANNEL FLOW PARAMETERS

- **Table 19.2**

Table 19.2 Hydraulic Parameters of Basic Channel Sections

section	area, A	wetted perimeter, P	hydraulic radius, R
 rectangle	dw	$2d + w$	$\frac{dw}{w + 2d}$
 trapezoid	$\left(b + \frac{d}{\tan\theta}\right) d$	$b + 2\left(\frac{d}{\sin\theta}\right)$	$\frac{bd\sin\theta + d^2\cos\theta}{b\sin\theta + 2d}$
 triangle	$\frac{d^2}{\tan\theta}$	$\frac{2d}{\sin\theta}$	$\frac{d\cos\theta}{2}$
 circle  circle	$\frac{1}{8}(\theta - \sin\theta)D^2$ [θ in radians]	$\frac{1}{2}\theta D$ [θ in radians]	$\frac{1}{4}\left(1 - \frac{\sin\theta}{\theta}\right) D$ [θ in radians]

GOVERNING EQUATIONS FOR UNIFORM FLOW

- Manning Equation

$$v = \frac{1.49}{n} R^{2/3} . S^{1/2} \quad \cdots \quad 19.12b$$

$$Q = A.v = \frac{1.49}{n} A.R^{2/3} . S^{1/2} \quad \cdots \quad 19.13b$$

v = velocity in ft/sec
Q = flow rate in cfs
R = hydraulic radius in ft
S = slope of energy gradient line (EGL) (dimensionless; not in %)
= slope of water surface (for uniform flow)
= slope of channel bottom (for uniform flow)
n = Manning roughness coefficient (Appendix 19.A)

- n varies with depth (Table 19.3, Appendix 19.C)

MANNING "n" TABLE

- Appendix 19.A
- Test: Find n for a concrete channel with rough joints
  - n = 0.016 – 0.017
- Higher the n value ⇒ higher the roughness ⇒ smaller the flow (∵ n in denominator)
- For sewers
  - n=0.013 (average value used in design)
  - n=0.015 (for old sewers)

APPENDIX 19.A
Manning's Roughness Coefficient[a,b]
(design use)

channel material	n
plastic (PVC and ABS)	0.009
clean, uncoated cast iron	0.013–0.015
clean, coated cast iron	0.012–0.014
dirty, tuberculated cast iron	0.015–0.035
riveted steel	0.015–0.017
lock-bar and welded steel pipe	0.012–0.013
galvanized iron	0.015–0.017
brass and glass	0.009–0.013
wood stave	
small diameter	0.011–0.012
large diameter	0.012–0.013
concrete	
average value used	0.013
typical commercial, ball and spigot rubber gasketed end connections	
– full (pressurized and wet)	0.010
– partially full	0.0085
with rough joints	0.016–0.017
dry mix, rough forms	0.015–0.016
wet mix, steel forms	0.012–0.014
very smooth, finished	0.011–0.012
vitrified sewer	0.013–0.015
common-clay drainage tile	0.012–0.014
asbestos	0.011
planed timber (flume)	0.012 (0.010–0.014)
canvas	0.012
unplaned timber (flume)	0.013 (0.011–0.015)
brick	0.016
rubble masonry	0.017
smooth earth	0.018
firm gravel	0.023
corrugated metal pipe (CMP)	0.024 (see App. 17.F)
natural channels, good condition	0.025
rip rap	0.035
natural channels with stones and weeds	0.035
very poor natural channels	0.060

[a]Compiled from various sources.
[b]Values outside these ranges have been observed, but these values are typical.

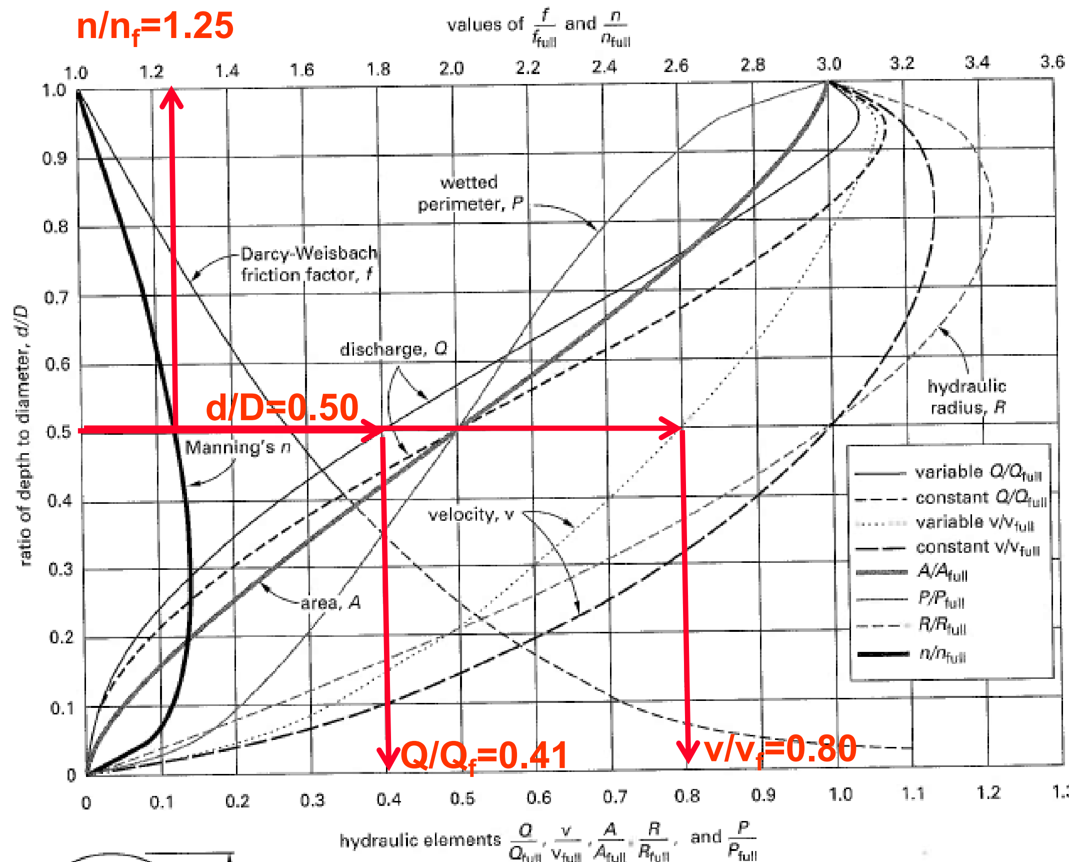
MANNING n VARIATION WITH DEPTH

APPENDIX 19.C
Circular Channel Ratios[a,b]

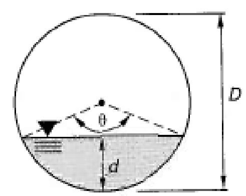
Experiments have shown that n varies slightly with depth. This figure gives velocity and flow rate ratios for varying n (solid line) and constant n (broken line) assumptions.

***Table 19.3** Circular Channel Ratios (varying n)*

$\frac{d}{D}$	$\frac{Q}{Q_{full}}$	$\frac{v}{v_{full}}$
0.1	0.02	0.31
0.2	0.07	0.48
0.3	0.14	0.61
0.4	0.26	0.71
0.5	0.41	0.80
0.6	0.56	0.88
0.7	0.72	0.95
0.8	0.87	1.01
0.9	0.99	1.04
0.95	1.02	1.03
1.00	1.00	1.00



At half depth, $Q=0.41 \times Q_{full}$
$v=0.80 \times v_{full}$



Governing equations

$$\theta_{deg} = 2 \arccos\left(\frac{\frac{D}{2} - d}{\frac{D}{2}}\right)$$

$$A = \left(\frac{D}{2}\right)^2 \frac{\theta_{rad} - \sin\theta_{deg}}{2}$$

$$P = \frac{D\theta_{rad}}{2}$$

$$R = \frac{A}{P}$$

$$v = \left(\frac{1.486}{n}\right) R^{\frac{2}{3}} \sqrt{S}$$

$$Q = Av$$

Slope is constant.

$n = 0.013$

$$\frac{n}{n_{full}} = 1 + \left(\frac{d}{D}\right)^{0.540} - \left(\frac{d}{D}\right)^{1.200}$$

$$\frac{n}{n_{full}} = 1 + \left(\frac{d}{D}\right)^{0.540} - \left(\frac{d}{D}\right)^{1.200}$$

EXAMPLE 19.2: Manning Equation

2.5 ft^3/sec (0.07 m^3/s) of water flows in a 20 in (0.5 m) sewer line (n = 0.015, S = 0.001). The Manning coefficient, n, <u>varies with depth</u>. Flow is uniform and steady. What are the velocity and depth?

- Given Data:
  - Fluid = water
  - Channel shape = circular
  - Diameter = 20 in
  - Flow rate = Q = 2.5 ft^3/sec
  - n = 0.015 (hint: use Manning Eq.)
  - n varies with depth
  - S = 0.001
  - Flow type = uniform and steady (hint: use Manning Eq.)

- Calculate (?):
  1. Flow velocity
  2. Depth of flow

EXAMPLE 19.2: SOLUTION

- From Eq. 16.19, R = D / 4 = (20/12) / 4 = 0.417 ft
- From Eq. 19.12(b)

$$v_{full} = \frac{1.49}{0.015}(0.417)^{2/3}(0.001)^{1/2} = 1.75\,\text{ft/sec}$$

$$Q_{full} = v_{full} \times A = 1.75 \times \frac{\Pi}{4}\left(\frac{20}{12}\right)^2 = 3.83\,\text{ft}^3/\text{sec}$$

$$\frac{Q}{Q_{full}} = \frac{2.5}{3.83} = 0.65$$

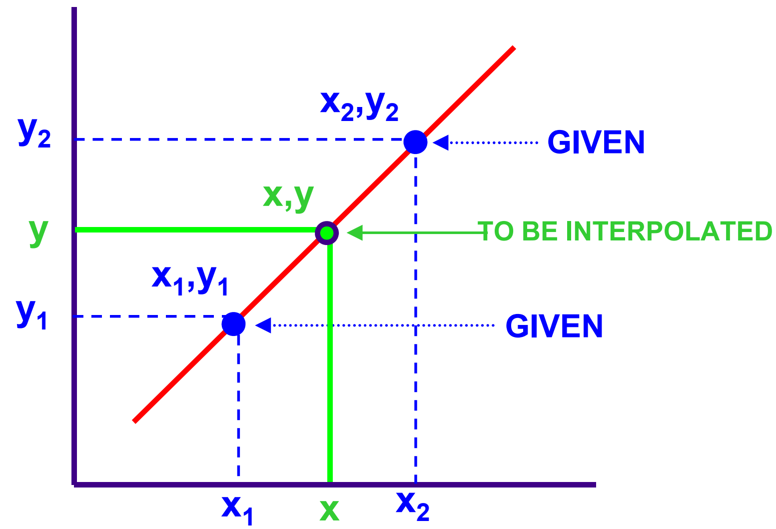
$\frac{d}{D}$	$\frac{Q}{Q_{full}}$	$\frac{v}{v_{full}}$
0.1	0.02	0.31
0.2	0.07	0.48
0.3	0.14	0.61
0.4	0.26	0.71
0.5	0.41	0.80
0.6	0.56	0.88
0.7	0.72	0.95
0.8	0.87	1.01
0.9	0.99	1.04
0.95	1.02	1.03
1.00	1.00	1.00

- From Table 19.3 using <u>interpolation</u>, for Q/Q_{full}=0.65
  - d/D = 0.66
  - v/v_{full}=0.92
- v = v_{full} x 0.92 = 1.75 x 0.92 = **1.61 ft/sec** ANSWER 1
- d = D x 0.66 = 20 x 0.66 = **13.2 in** ANSWER 2

HOW TO INTERPOLATE: PROGRAM YOUR CALCULATOR

Interpolate between
(x_1,y_1) and
(x_2,y_2)
to determine (x,y)

- **Interpolation equation**



$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

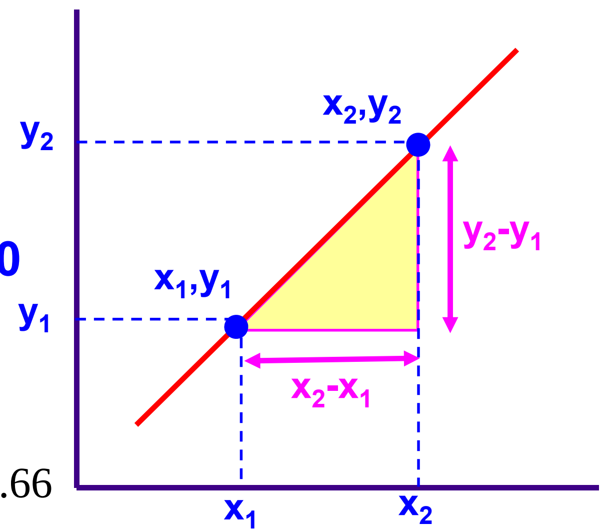
$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

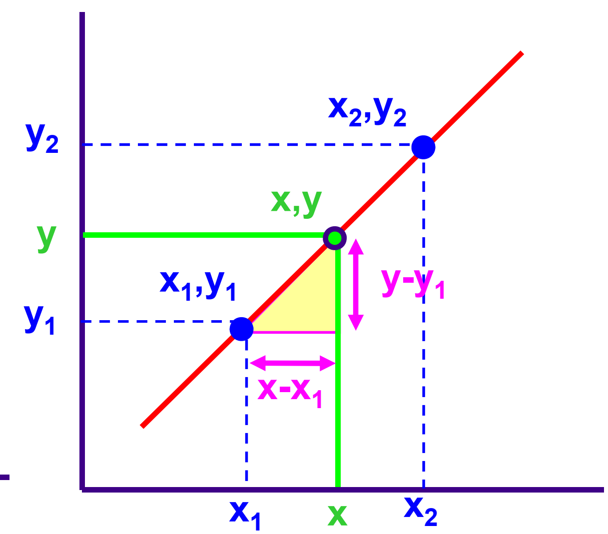
$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Can be programmed into programmable calculators
Great for FE/PE exams

- **In Ex. 19.2: $x_1=0.56$, $y_1=0.60$**
  $x_2=0.72$, $y_2=0.70$
  $x=0.66$, $y=?$

$$y = 0.60 + \frac{0.70 - 0.60}{0.72 - 0.56}(0.66 - 0.56) = 0.66$$





FROUDE NUMBER AND FLOW REGIMES

- Dimensionless Froude number, Fr, is a convenient index of flow regime (subcritical or supercritical flow)

$$Fr = \frac{v}{\sqrt{gL}} \qquad \cdots \quad 19.78$$

v = velocity

$$L = \text{characteristic length} = \text{mean hydraulic depth} = D_h = \frac{A}{W}$$

$$Fr = \frac{v}{\sqrt{g D_h}}$$

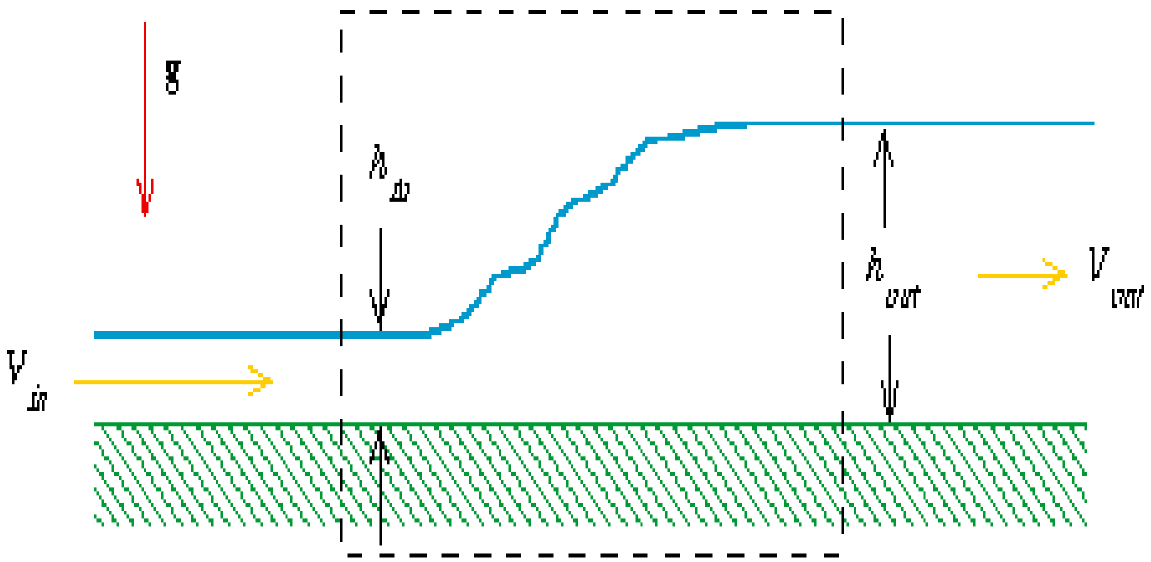
- When Fr < 1, flow is subcritical
  - Depth of flow > critical depth
  - Velocity < critical velocity
- When Fr > 1, flow is supercritical
  - Depth of flow < critical depth
  - Velocity > critical velocity
- When Fr =1, flow is critical

HYDRAULIC JUMP

- Occurs when
  - High velocity flow (supercritical) meets slow-moving (subcritical) flow.
  - There is a sudden transition from supercritical ($y < y_c$) to subcritical ($y > y_c$) flow.
- Velocity is reduced rapidly over short length of channel
- The abrupt rise in the water surface is known as a "hydraulic jump."

HYDRAULIC JUMP

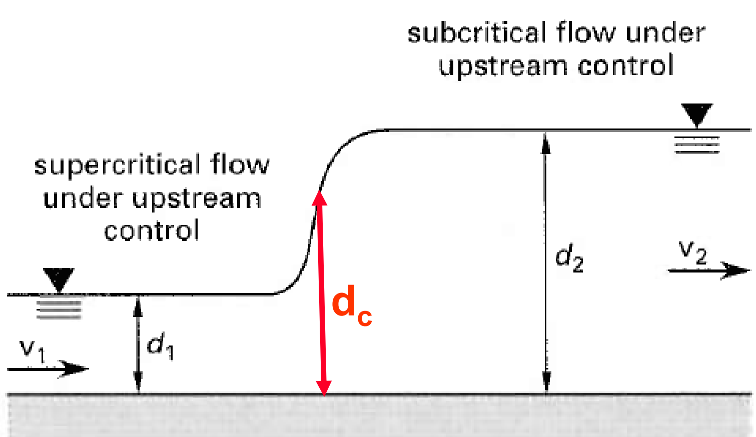
- The increase in depth is always from below the critical depth to above the critical depth
- Examples of where this may occur are :
  - At the foot of a spillway
  - Where a channel slope suddenly turns flat



HYDRAULIC JUMP EQUATIONS

- The depths d_1 and d_2 on either side of hydraulic jump are known as conjugate depths.
  - d_1 = Initial depth < d_c (supercritical depth).
  - d_2 = Sequent depth > d_c (subcritical depth)

Figure 19.21 Conjugate Depths



$$d_1 = -\tfrac{1}{2}d_2 + \sqrt{\frac{2v_2^2 d_2}{g} + \frac{d_2^2}{4}} \quad \left[\begin{array}{c}\text{rectangular}\\ \text{channels}\end{array}\right] \qquad 19.90$$

$$d_2 = -\tfrac{1}{2}d_1 + \sqrt{\frac{2v_1^2 d_1}{g} + \frac{d_1^2}{4}} \quad \left[\begin{array}{c}\text{rectangular}\\ \text{channels}\end{array}\right] \qquad 19.91$$

$$\frac{d_2}{d_1} = \tfrac{1}{2}\left(\sqrt{1 + 8(\mathrm{Fr}_1)^2} - 1\right) \quad \left[\begin{array}{c}\text{rectangular}\\ \text{channels}\end{array}\right] \qquad 19.92(a)$$

$$\frac{d_1}{d_2} = \tfrac{1}{2}\left(\sqrt{1 + 8(\mathrm{Fr}_2)^2} - 1\right) \quad \left[\begin{array}{c}\text{rectangular}\\ \text{channels}\end{array}\right] \qquad 19.92(b)$$

If the depths d_1 and d_2 are known, then the upstream velocity can be found from Eq. 19.93.

$$v_1^2 = \left(\frac{g d_2}{2 d_1}\right)(d_1 + d_2) \quad \left[\begin{array}{c}\text{rectangular}\\ \text{channels}\end{array}\right] \qquad 19.93$$

HYDRAULIC JUMP EQUATIONS

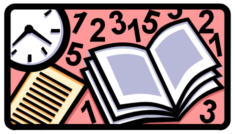
- **Approximate lengths of hydraulic jumps in Table 19.6**

Upstream Froude No. $N_{Fr,1}$	L/d_2
3	5.25
4	5.8
5	6.0
6	6.1
7	6.15
8	6.15

- **The energy loss in a hydraulic jump can be found by Eq. 19.94**

$$\Delta E \approx \frac{(d_2 - d_1)^3}{4 d_1 d_2} \qquad \cdots 19.94$$

PRACTICE PROBLEM: HYDRAULIC JUMP

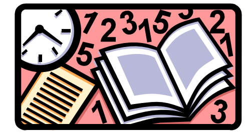


If 12 m^3/sec of water per meter of width flows down a spillway onto a horizontal floor and the velocity is 20 m/sec, determine (a) the downstream depth required to cause a hydraulic jump, (b) the loss in energy head, and (c) the losses in power by the jump per meter of width (Example 3.18, Streeter, et al.)

- Given Data:
  Flow per unit width = q = 12 m^3/sec per meter
  v_1= 20 m/sec

- Calculate (?):
  - d_2
  - ΔE
  - Power loss per unit width

PRACTICE PROBLEM: SOLUTION



- Q = A x v
- Divide each side by w to obtain flow per unit width "q"
  q = d x v
  d = q / v
- $d_1 = q / v_1 = 12 / 20 = 0.6$ m
- From Eq. 19.91,

$$d_2 = -\frac{1}{2}0.6 + \sqrt{\frac{2(20^2)(0.6)}{9.806} + \frac{0.6^2}{4}} = \boxed{6.7m}$$

ANSWER 1

- From Eq. 19.94,

$$\Delta E = \frac{(6.7 - 0.6)^3}{4(0.6)(6.7)} = \boxed{14.1\ m}$$

ANSWER 2

- From Table 18.6, Power (kW) = 9.81 x head x flow
  = Power loss (kW) / width = 9.81 x head loss x (flow/width)
  = 9.81 x 14.1 x 12 = **1659.85 kW/m** ANSWER 3

HYDRAULIC JUMP

- Section from course CD-ROM
- Conjugate depths ≠ alternate depths
- At alternate depths, specific energy is constant
- At conjugate depths, specific energy is not constant

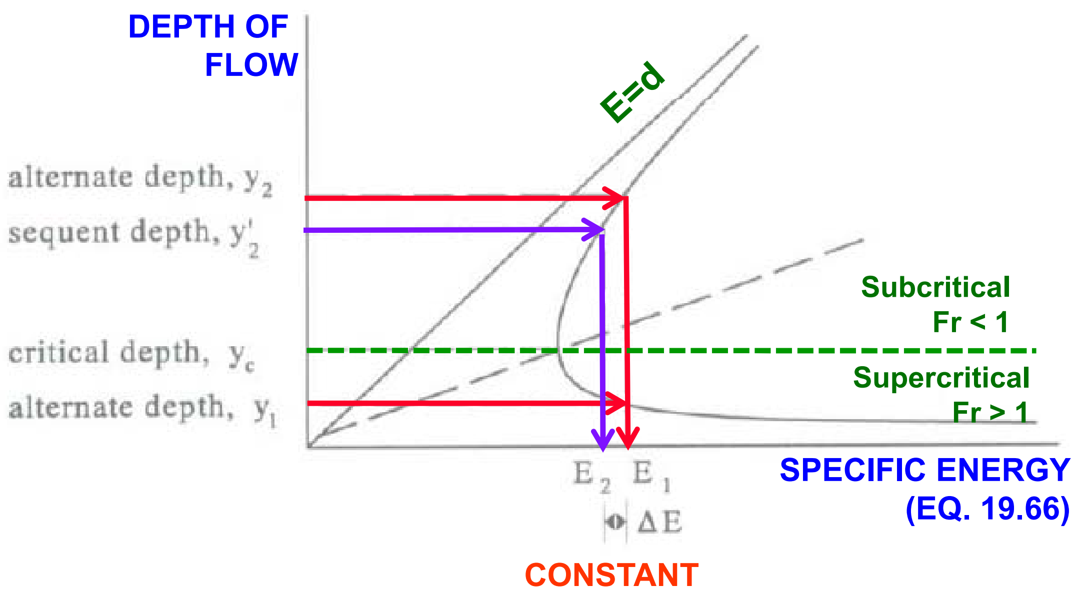
3. Hydraulic Jump

Hydraulic jump occurs when depth of flow occurs quickly from low stage to high stage in a channel. The depth before the jump is always less than the depth after the jump.

Depth before the jump: initial depth, y_1
Depth after the jump: sequent depth, y'_2

Sequent depth, $y'_2 \neq y_2$, alternate depth, because with alternate depths specific energy is constant, however, with sequent depth an energy loss occurs

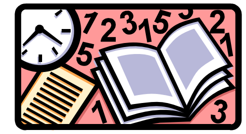
SPECIFIC ENERGY DIAGRAM (FIG. 19.13)



Hydraulic jump will not occur if the downstream flow is supercritical. In addition to dam spillways, hydraulic jump is often used in storm culvert design to control the outlet energy. The length of a hydraulic jump determines the minimum length of channel protection needed, see Table 5.4 in Lindeburg. **(Now Table 19.6)**

In order to provide the most favorable protection for natural channels, the downstream velocity is usually limited to a set maximum, such as 3 fps in soil.

PRACTICE PROBLEM 19-14: HYDRAULIC JUMP FROM PRACTICE PROBLEMS BOOK



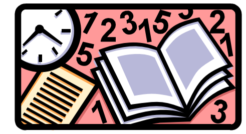
Water flowing in a rectangular channel 5 ft wide experiences a hydraulic jump. The depth of flow up-stream from the jump is 1 ft. The depth of flow down-stream from the jump is 2.4 ft. What quantity is flowing?

A. 39 ft^3/sec
B. 45 ft^3/sec
C. 52 ft^3/sec
D. 57 ft^3/sec

- Given Data:
  $w = 5$ ft
  $d_1 = 1.0$ ft
  $d_2 = 2.4$ ft

- Calculate (?):
  - Q

PRACTICE PROBLEM 19-14: SOLUTION



- From Eq. 19.93,

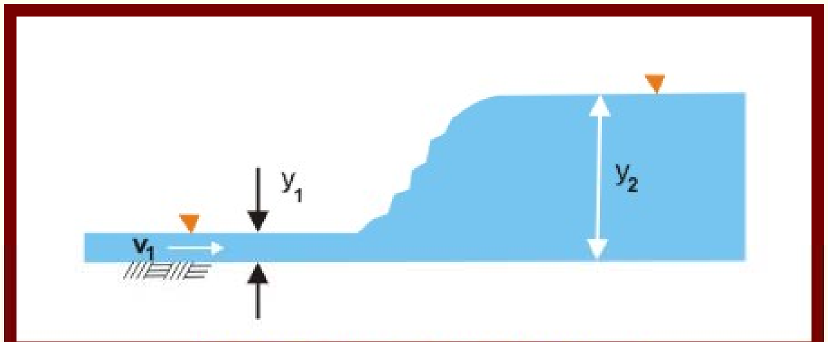
$$v_1 = \sqrt{\frac{gd_2}{2d_1}(d_1 + d_2)} = \sqrt{\frac{32 \times 2.4}{2 \times 1}(1 + 2.4)} = 11.46 \text{ ft/sec}$$

- Q = A x v_1 = (w x d_1) x v_1
  = (5 x 1.0) x 11.46
  = **57.30 ft^3/sec**

ANSWER – SELECT D

PRACTICE PROBLEM 19-14: WEB CHECK BACK CALCULATE D2 USING CALCULATED Q

onlinechannel11.php: Calculation of the sequent depth [y_2] of a hydraulic jump



The hydraulic jump in a horizontal channel

Formulas:

$$F_1 = v_1/(gy_1)^{1/2}$$

$$q = v_1 y_1$$

$$y_2 = (y_1/2)\ [(1 + 8F_1{}^2)^{1/2} - 1]$$

$$v_2 = q/y_2$$

$$F_2 = v_2/(gy_2)^{1/2}$$

INPUT DATA:

Select: SI units (metric) / U.S. Customary units

Flow depth y_1: 1 ft

Flow velocity v_1: 11.46 fps

GIVEN DATA

INTERMEDIATE CALCS:

Units selected: U.S. Customary

Grav. accel. [g]: 32.17 ft s^{-2}

Discharge q: 11.46 cfs ft^{-1}

Froude number F_1: 2.020

OUTPUT:

Flow depth y_2: 2.400 ft ANSWER

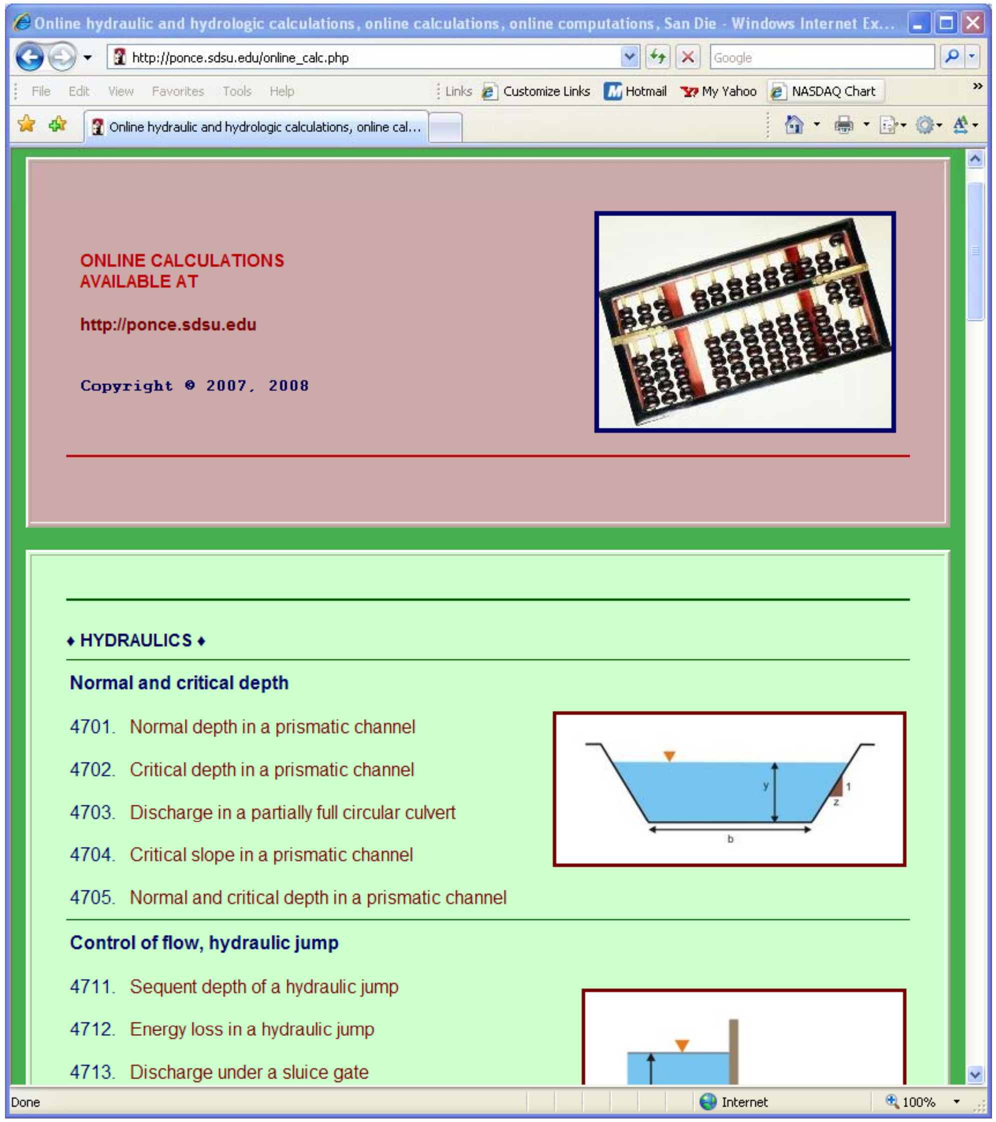
Flow velocity v_2: 4.773 fps

Froude number F_2: 0.543

Press button to Calculate **or recalculate**

http://ponce.sdsu.edu/onlinechannel11.php (Link good as of 11/4/09)
Prof. Victor Miguel Ponce's Web site, San Diego State University

ONLINE HYDROLOGIC AND HYDRAULIC CALCULATOR



11/4/09 Link: http://ponce.sdsu.edu/online_calc.php
Prof. Victor Miguel Ponce's Web site, San Diego State University

SUGGESTED READING: CHAPTER 19

- 12. Most efficient cross section
- 14 to 16. Weirs
- 37-39. Culverts